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# Riemann and Mobius dust the Kakeya Problem

Thierno M. SOW\*

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*“Nihil superest cuius oppositum tu credis.”*

G. Cardano.

## Abstract

The purpose of this article is to release an elegant proof of the Kakeya needle problem for any  $n$ -dimensional space, in the light of the Riemann Hypothesis theorem and the Listing-Mobius ring.

Mathematics Subject Classification 2010 codes: Primary 11M26; Secondary 42B25

## 1 THE KAKEYA NEEDLE PROBLEM

One hundred years ago, **Soichi Kakeya** asked whether there is a minimum area of a region  $\mathbf{D}$  in the plane in which a needle of unit length can be turned through 360 degrees.

## 2 WHAT IS NEW?

All sciences lead to Riemann and the most elegant proof of the Riemann Hypothesis can be expressed as follows:

**Theorem 1.** *There are infinitely many nontrivial zeros on the critical line and all these zeros have real part  $1/2$ .*

*Proof.*

$$\prod_{k=1}^{\infty} \left( \frac{1}{2} + \frac{1}{p_k^s - 1} \right) = \zeta(s), \quad (1)$$

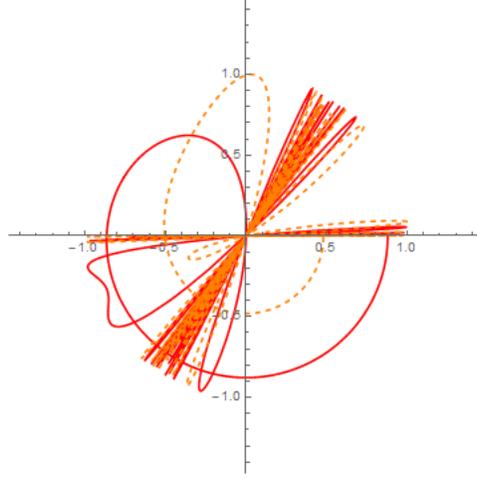
where  $s \in \mathbb{C}$ , and  $p \in \mathbb{P}$ .

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We can observe the perfect zeta function, valid on the whole complex plane, with a convergent series represented by the following polar.



**Algorithm 1.** Code Mathematica

```
PolarPlot[{{Cos[(t^t + 1) / (2 t^t - 2)]},
Sin[(t^t + 1) / (2 t^t - 2)]}, {t, -2π, 2π},
PlotStyle->{Red,Directive[Dashed,Green,Orange]}, PlotRange-> All]
```

Now, the question is how to connect the Riemann hypothesis theorem to the Kakeya needle problem? We start the sketch of the proof by observing that we can make the following assumption. Let us consider  $n$  as the area of the given figure. We may also assume  $p$  is contained in the unit figure such that the tip of the needle corresponds to the smallest prime factor of  $n$ . It follows:

**Theorem 2.** *The minimum area of a region  $\mathbf{D}$  in the plane in which a needle of unit length can be turned through 360 degrees is zero.*

*Proof.*

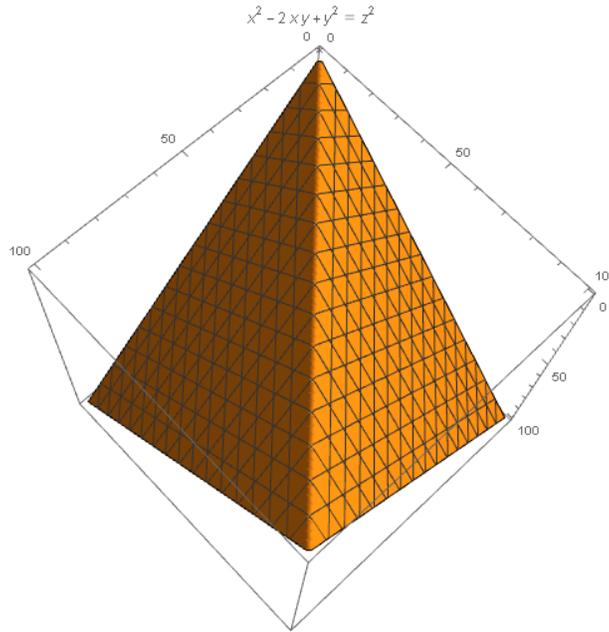
$$\prod_{k=1}^{\infty} \frac{p_k^s + 1}{2(p_k^s - 1)} = \zeta(s) = \sum_{n=1}^{\infty} \frac{n^2 - p^4}{p^2} + \frac{n^2 - q^4}{q^2}, \quad (2)$$

where  $n \equiv 0 \pmod{p}$ ,  $n \equiv 0 \pmod{q}$  and  $p_k$  denotes the  $k$   $n$ -th prime.

It is specially true for all cases for which the Kakeya set corresponds to a *prime*-dimensional unit. Furthermore, for the general case, to obtain the precise value of the RHS of the Riemann zeta function we assume, a mere glance (the previous article) will show that, under suitable conditions

$$\frac{p^s + 1}{2(p^s - 1)} = \frac{1}{2} + \frac{1}{p^s - 1}. \quad (3)$$

To illustrate the situation, we want to recall a simple geometrical fact which was already used for  $x = p$ ,  $y = q$  and  $z = q - p$ .



Algorithm 2. Code Mathematica

```
ContourPlot3D[x^2 + y^2 - 2 x y == z^2, {x, 1, 100}, {y, 1, 100}, {z, 1, 100},
PlotLabel-> x^2 + y^2 - 2 x y == z^2]
```

### 3 WHAT IS ALREADY?

#### 3.1 THE HORUS EYE THEOREM

We assume: *by the properties of the light-sensitive cells, every image or object is inverted on our retina.* This input-output relation corresponds to a quantum informations system which has deep connections with the field of mathematics of what has come to be known as geometric measure theory. According to some specialists in Optics like D. L. Anderson: “*the size of the image projected on the retina is determined by the size of the object and the distance of the object from the perceiver...two objects of the same height will not project the same size image on the retina if one of them is further away than the other.*” Which means that the smallest Kakeya set depends more on the distance and remains zero, at least the set corresponds to the smallest length of a quantum light-sensitive cell. In times past, the Egyptians knew that the truly structure of the Horus eye corresponds to the set  $\frac{1}{n^s}$ .



### 3.2 THE KAKEYA-LISTING-MOBIUS THEOREM

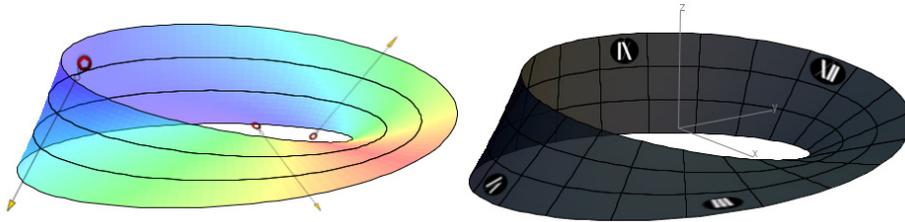
**Theorem 5.** *The minimum area of a region  $D$  in any  $n$ -dimensional space in which a needle of unit length can be turned through 360 degrees is zero.*

*Proof.*

$$\mu(n) = \begin{cases} 1 & \text{if } n = 1 \\ (-1)^k & \text{if } n \text{ is the product of } k \text{ } n\text{-th distinct primes} \\ \zeta(s) & \text{otherwise.} \end{cases} \quad (4)$$

*The results confirm the proof by the Riemann Hypothesis theorem since for all cases in which  $n$  is prime  $\mu(n) = \zeta(s) = 0$ .*

Ultimately, we can observe if a needle were to slip along the length of the “fancy clock” below, it would turn through 360 degrees having just traversed a short part-betwixt of the entire length of the ring without crossing an edge. Therefore, it is *needless to wind up the Kakeya Clock*. Namely, we improved successfully the results to any  $n$ -dimensional space and we doubled the number of 360 degrees tour inside, which makes the proof more *elegant*.



In troth, the old paths have joined the new.



## 4 MISCELLANEOUS

### 4.1 THE HOKUSAI THEOREM

In this section, would we introduce some masterpieces sank into oblivion which are truly connected to the Kakeya needle problem and the Riemann Hypothesis. First of all, let us recall the fact that the Japan has always had great mathematicians as well as great artists like the Haiku master Matsuo Basho (1644 – 94), Harunobu (1725 – 70) and his painting “*Two Under an Umbrella*”, Ando Hiroshige (1797 – 1858), and the famous master Hokusai (1760 – 1849), among others. Indeed, two weeks ago, we were writing an article on, one of our favourite Japanese authors, Yoshikichi Furui when it has been brought to our attention that the Kakeya problem fulfilled perfectly the secret meanings of another Hokusai masterpiece “*The Barrel-maker of Fujimihara*” (c.1830).



At first, for many art historians, it was thought that the tale of this scene is about the life of a slender whacked man slanting the *Nemesis*'s wheel of *fortuna* in opposite to the lyrical and charming view of the landscape behind him. Maybe! But, for the mathematicians who have a deep knowledge of the Art, let us say, there are wheels within wheels. Indeed, Mathematics play a central role in the Japanese Arts and Culture as well as in the Architecture. One can see an example of beautifully balanced asymetry in the plan of the Asukadera temple or the plan of Fujiwarakyo (694 – 710), an Imperial City. Likewise, in Geometry the Japanese art of paper folding is well-known as Origami and the Japanese taste for Mathematics and the prime numbers can be seen in the seventeen-syllable Haiku or the thirty-one-syllable humorous form called *Kyoka*. An excellent example of their sophistication is the following thirty-one-syllable love poem by one Princess Kagami at the court of Emperor Tenji (r.662 – 72):

Like the hidden stream  
trickling beneath the trees  
down the mountainside  
so does my love increase  
–more than yours, my Lord.

*Ian Hideo Levy, Ten Thousand Leaves, Princeton.1981.vol.2.92.*

To investigate further one can find out at the Library of Congress the *History of Japanese Art* by P. E. Mason (1993) and revised by D. Dinwiddie (2003).

Talking about Mathematics in the art of the master Hokusai, we was intrigued and aroused by the similar canvas and ceilings, like a keymaster in Cryptography, we both used. Indeed, in the painting below “*The Well-Guided and the Arch of the Primes*”, we have tried to represent the modular frequency distribution of the primes (the wheels) along to the Riemann zeta function (the lighthouse) and the light comes out when the sieve revealed that the perspective of the wave inside the wheel is exactly the same than the master Hokusai used to depict the Mount Fuji and the character inside the disk of the barrel. Now, what does it means mathematically?



We do know that the Mount Fuji represents the immortality (infinity) and as a child Hokusai was fascinated by the mirrors, so, if *Hokushinsai* means the sparkle of star’s room then it follows that the Hokusai problem can be expressed as follows: *the infinite volume of any smooth surface whithout any thickness is paintable with a finite amount (coats) of paint because  $\zeta(s) \rightarrow 0$  when  $n \rightarrow +\infty$ .* Exactly what Hokusai did in his whole life, painting “infinitely” many times the Mount Fuji with finitely many numbers of coats of paint. Likewise, in Mathematical modeling, there exists deep connections between the Gabriel’s horn paradox and the Riemann zeta function (more, see M. Lynch, and M. Henrikson -1965-).

For instance one can see an hint about the Euler product in the volume of the Gabriel's horn

$$V = \pi \int_1^p \left(\frac{1}{n}\right)^2 dn = \pi \underbrace{\left(1 - \frac{1}{p}\right)}_{\text{the Euler seal}}. \quad (5)$$

So, Hokusai knew that the character and the Mount Fuji can be turned through 360 degrees inside the barrel as the Kakeya needle will do if the area of the Listing-Mobius ring equals to zero.

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## 5 WHAT IS NEXT?

In the next article we will study the score of the music of the primes before to release the Next **Prime** Theorem and we will see the consequences of the Riemann Hypothesis theorem on the General Theory of Relativity and how Bernhard Riemann changes definitely the interplay between the new Quantum Number Theory and Modern Physics.

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