Riemann Shatters The Gordian Knot

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"*Riemann plays dice.*"

Abstract

Solving the soaring Riemann Hypothesis is equivalent to face the overawing challenge of Alexander the Great to slicing up or to unravel the conundrum of the Gordian Knot. In this article, we will discuss about the sufficient conditions to prove the Riemann Hypothesis. Likewise, we shed the light on some related problems in number theory and Physics. Thereby, we will vet the proof by revealing how many conjectures are sewn on the edges of the Riemann Hypothesis like the abc-conjecture, the twin prime conjecture, the prime number theorem, the Legendre conjecture and beyond. Ultimately, what is peculiar in our research is the one sentence proof.

Mathematics Subject Classification 2010 codes: Primary 11M26; Secondary 11A41

1 THE RIEMANN HYPOTHESIS PROOF

All sciences lead to Riemann and the most elegant proof of the Riemann Hypothesis can be expressed as follows: *there are infinitely many nontrivial zeros and they all have real part* $1/2$.

The gleaming Proof.

$$\int_{\mathbb{C}} \int_{\mathbb{P}} \left(\frac{1}{2} + \frac{1}{i^2 + \log{(p)}^{s\pi}} \right) dsdp = \zeta(s), \tag{1}$$

$$\textit{where } i^2 = -1, \ s \in \mathbb{C} \textit{ and } p \in \mathbb{P}.$$

■

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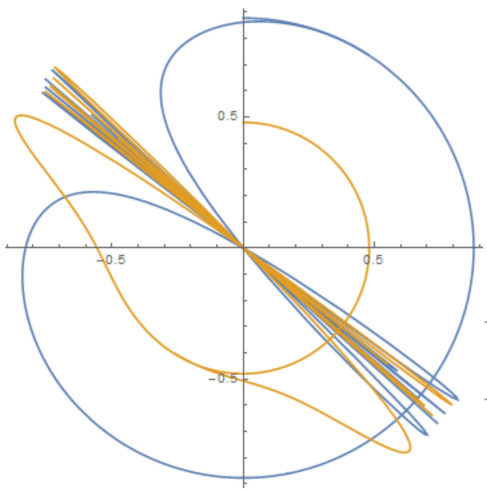
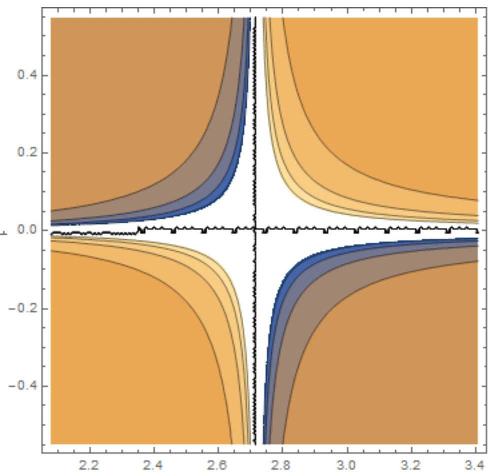
We can observe the “apple” and the perfect symmetry in the following figures. A seasoned physicist will see immediately, in the polar, the similar gleam floating on the top of a Black Hole and the Superellipse with concave sides.





Algorithm 1. Code Mathematica

```
PolarPlot[{{Cos[1/2 + (-1 + t Log[t]^(π t))^(-1)]},
Sin[1/2 + (-1 + t Log[t]^(π t))^(-1)]}, {t, −2.5π, 2.5π},
PlotStyle->{Red,Directive[Dashed,Green,Orange]}, PlotRange-> All]
```

2 THE RIEMANN HYPOTHESIS: THE GIST

In his celebrated article “*On the number of primes less than a given magnitude*”, released in 1859, Bernhard Riemann (1826 – 1866) depicted the importance of his “*investigation into the accumulation of the prime numbers; a topic which perhaps seems not wholly unworthy of such a communication, given the interest which Gauss and Dirichlet have themselves shown in it over a lengthy period*”. In troth, since then, no one didn’t provide the neatest statement of what the Riemann Hypothesis is or what the Riemann Hypothesis is not. By the way, Riemann himself was very careful and didn’t provide any sufficient raw material, his article has just 8 pages. Indeed, before to tackle the problem we need to catch the philosophy behind. Since the prime numbers were discovered, Mathematicans have been always fascinated by their mysterious distribution. Having utterly failed in the pursuit of the grail, they reformulate the next prime conjecture into the prime number conjecture, whilst their goal was to settle the gobal pattern behind the distribution of the prime numbers. They said, if we are not able to determine whether a given integer is prime and then what is the next prime, we can at least reduce the problem by substitution. Which means, if the pattern of the distribution of the primes is printable on canvas, then, there exists a function on the accumulation of the prime numbers given a certain quantity. Therefore, counting the number of primes is not the premium goal of the Riemann Hypothesis but rather the cunning maneuver displayed by Riemann to handle a more important issue. For instance, computing the number of primes above a given threshold might be out of reach, in 1859. Indeed, according to Gauss: "*the problem of distinguishing prime numbers from composite numbers and of resolving the latter into their prime factors is known to be one of the most important and useful in arithmetic...Further, the dignity of the science itself seems to require that every possible means be explored for the solution of a problem so elegant and so celebrated*". This is why, the Riemann Hypothesis may be defined as the Swiss Knife of the number theory, and may be considered as the most difficult problem in Mathematics because many conjectures are fettered with the Riemann Hypothesis. So, what is knew?

Bernhard Riemann has introduced three great breakthroughs at the forefront of major advancements in number theory. The first is **the critical line**, which means that in the non-Euclidean "modern" number theory a line is not necessarily straight and it passes through one coordinate. The second is the **nontrivial zeros** and **their real part** which are useful, to determine whether a given integer is prime and, at the same time, to prove that there are infinitely many primes. At last, to obtain the perfect symmetry, the negative values of the nontrivial zeros should depend solely on **the complex number** s.

According to those criteria we define the gist of the Riemann Hypothesis as follows: *there are infinitely many nontrivial zeros which all have real part* $1/2$.

3 THE INGREDIENTS TO WIELD

In this section we will discuss about the sufficient conditions to prove the Riemann Hypothesis. First, let us pinpoint some confusing statements purpoted by many people about the Riemann zeta function. The following identity inspired by Euler is not the Riemann zeta function but a pretty good knack that Riemann had picked up to ply the problem.

$$\prod_{p\in\mathbb{P}}^{\infty}\frac{1}{1-\dfrac{1}{p^s}}=\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}. \tag{2}$$

It tells us that the Riemann zeta function may satisfy a relation between the product over the primes, in the left hand side, and, the infinite sum of natural numbers, on the right hand side, without knowing nothing about the prime numbers. It turns out, that this function has trivial zeros for some negative integers. Ultimately, we should consider this function as the preliminary rough outline of a master. Let us call it the Euler-Riemann Identity. Otherwise, here we are dealing with complex numbers on the form $\dfrac{1}{2}+\epsilon$, which means that computing $\zeta(2)$, $\zeta(3)$ and so on, is as asinine as counting the nontrivial zeros.

Also, the complex variable χ introduced by Dirichlet in 1837 has nothing to do with the Riemann complex number s. But a question remains because, in some special cases, $s = 0$ doesn't make any sense, even if obviously 0 is a complex number. For instance, Euler knew that $\zeta(0) = -1/2$.

The second important point refers to the valuable region of the plane. Unlike what has been reported, the Riemann zeta function is not only defined in the half-plane $\Re(s) > 1$ but in the whole complex plane. The goal of our last point is twofold. Indeed, whatever the form or the identity of your equation it should embed two main functionalities: the primality test or the factorization for any given integer in $O(\log(n))$ and the prime counting function for any given quantity $\pi(N)$. The justification is given by the fact that, we assume: *there are many functions satisfying the Euler-Riemann Identity.*

Proof.

$$\prod_{k=1}^{\infty} \frac{s}{2\left(p_k + i^2\right)} = \zeta(s) = \sum_{n=0}^{\infty} \frac{1}{2} p^n (i^2) s, \tag{3}$$

where $i^2 = -1$*,* $s \in \mathbb{C}$ *and* $p_k \in \mathbb{P}$ *denotes the* k*-nth prime.*

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We will observe further, the deep connections between the abc-conjecture proof and the relation above.

Those criteria put the steepness of the slope at the highest magnitude. That's just the way it is. Loosely, the Gordian Knot is not the *Herakleotikon Hamma*. Isn't it?

4 HOW TO VET THE PROOF

To vet the Riemann Hypothesis Proof, we release the official check list such that the neatest proof poised on the following points:

1. Does your equation is useful to determine whether a given integer is prime?
2. Does your equation embeds a prime counting function for a given quantity?
3. Does your equation generates the next prime?
4. Does the symmetry in the complex plane depends on the complex number?
5. Does your equation implies a generalization of the real part?

The purpose of the points 4 and 5 is to hone the generalization of the Riemann Hypothesis such that you can move and translate the critical line all over the complex plane. This functionality is very useful in Modern Physics.

Thereupon, your equation generates infinitely many nontrivial zeros which all have real part $1/2$. *Congratulations! Your mission has been successfully accomplished!*

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5 WHETTING THE EDGES OF THE PROOF

There are infinitely many nontrivial zeros and they all have real part $1/2$.

Proof.

$$\prod_{p\in\mathbb{P}}^{\infty}\frac{1}{2-\dfrac{2}{p^s}}=\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{(2n+2)^s}, \tag{4}$$

where $s=\dfrac{p^2+1}{4}$ on the LHS and $s=\dfrac{n^2+1}{4}$ on the RHS such that

$$\lim_{p\to\infty}\frac{1}{2-2p^{\frac{1}{4}(-1-p^2)}}=\frac{1}{2}=\lim\sum_{n=1}^{\infty}\frac{1}{(2n+2)^{\frac{1}{4}(1+n^2)}}. \tag{5}$$

Algorithm 2. Code Mathematica

```
NSum[1/(2n+2)ˆ((nˆ2+1)/4),{n,1,Infinity}]
```

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Now we elaborate the proof hereinafter.

5.1 THE FIRST FLOOR

Here we are. To start the sketch of the proof we recall a very useful relation. If $n=pq$ where p and q are primes then, $p^2+q^2-2n=(q-p)^2$, where $p<q$. We can improve this straightforward geometrical relation such that

$$\zeta(s)=s\left(\sum_{n=1}^{\infty}\frac{n^2-p^4}{p^2}+\frac{n^2-q^4}{q^2}\right)=0. \tag{6}$$

This means that factoring implies also additive properties. We will observe the same phenomena in the abc-conjecture proof. For now, we can conclude that every integer n has at least a complex number s on the form

$$\int\frac{n^2+\alpha^4}{4\alpha^2}d\alpha=s=\frac{1}{2}+it, \tag{7}$$

where α denotes every factor of n, i denotes the imaginary part and t is the complex variable introduced by B. Riemann himself.

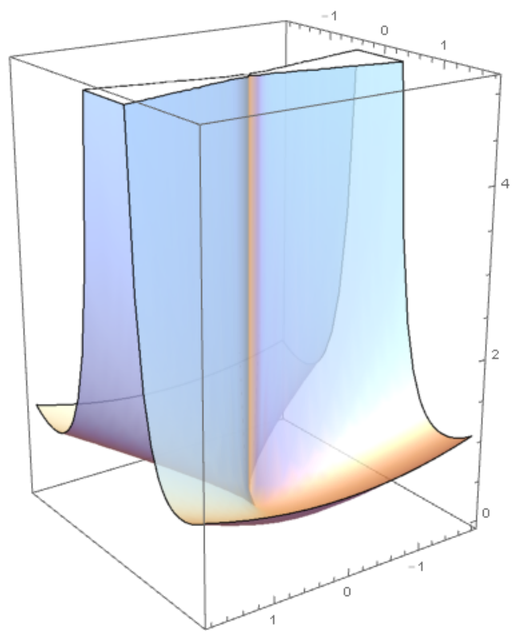
For instance if $7.11.23=n=1771$ then, $s_7=\dfrac{1}{2}+16014$, $s_{11}=\dfrac{1}{2}+6510$ and $s_{23}=\dfrac{1}{2}+1614$. It's interesting to observe how s_7 and s_{23} are peculiar.

It yields that, we assume: *a prime number is an integer greater than one which has a unique coordinate on the critical line on the form*

$$\frac{p^2+1}{4} = s_p = \frac{1}{2} + it. \tag{8}$$

Recall, the purpose is not just to prove if there are infinitely many zeros between $1/2$ and 1. Namely, every prime is fettered with a single nontrivial zero which embeds a real part a half, that is the Riemann Hypothesis!

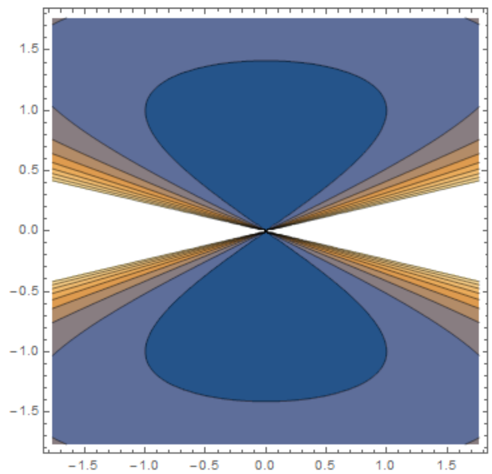
So far, we can observe the perfect symmetry in the following figure:



Algorithm 3. Code Mathematica

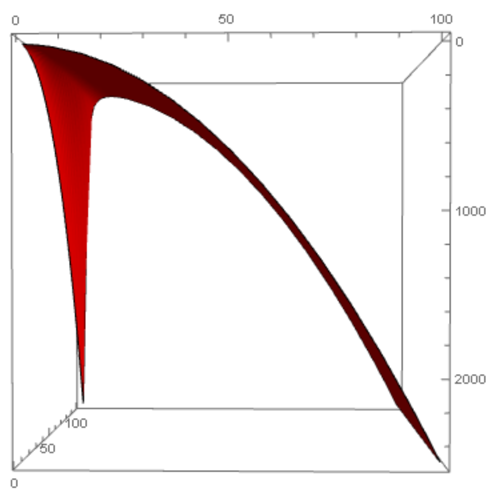
```
Plot3D[(n^2 + x^4)/(4 x^2), {n, -1.8, 1.8}, {x, -1.8, 1.8}, Mesh->None,
PlotStyle->Directive[Red,Specularity[White,20],Opacity[0.8]],
ExclusionsStyle->{None,Red}, BoxRatios->Automatic]
```

We can also observe the perfect convergence.



Algorithm 4. Code Mathematica

```
ContourPlot[(n^2 + x^4)/(4 x^2), {n, -1.76846, 1.76846}, {x, -1.76846, 1.76846}]
```



Algorithm 5. Code Mathematica

```
Manipulate[Plot3D[(n^2 + x^4) / (4 (x^2)), {n, 1, 100}, {x,1,100},
ColorFunction->(Red &), Mesh->None, BoxRatios->1]]
```

This is the most important breakthrough in the RSA encryption. Indeed, factoring is now feasible in $O(\log(n))$, since for every $n = pq$ where p and q are primes, there exists a unique complex number. Namely, you don't need to build a table of primes and you don't need anymore to divide n by the primes less than n. For instance, let $n = 1337$ so, you can find 7 and 191, simply by plugging the complex number $s_{p,q} = \frac{1}{2} + 9132$.

Of course, we know how to find the complex variable for any RSA number, but for security reasons, in a fickle world, we will not put the complete statement on a public wall. In the other hand, keep in mind, the RSA encryption is known to be billion times harder to crack than the Enigma machine, i.e.. So, you can call it **The Secret Theorem** and feel free to find the solution by yourself. Hint: the sieve may be modular in $\equiv 5 \pmod{10^{\theta/2}}$ where θ depends on the scale and the number of digits of n.

Fortunately, the Riemann Hypothesis proof implies the smartest encryption for the next generations.

1. Does your equation is useful to determine whether a given integer is prime?

YES!

■

5.2 THE SECOND FLOOR

We shall write and define $\Re(p)$ for the Riemann counting function for the number of primes less than p. At the first sight, we might think that $\Re(p)$ and $\pi(N)$ are slightly different. Let $\beta \in \mathbb{R}$, it follows:

$$\Re(p) = \frac{p^2 + i^2}{4?} \pm \beta. \tag{9}$$

For instance

$$6 = \Re(13) = \frac{42}{7} = \pi(13), \tag{10}$$

and

$$169 = \Re(1009) = 1 + \frac{254520}{1515} = \pi(1009). \tag{11}$$

Since $\Re(p)$ holds for a given prime p, so, let us merge the both equations such that

$$\Re(p) = p^2 + 4\alpha(\beta - \pi(N)) + i^2 = 0. \tag{12}$$

We assume: *there will always exist* $(\alpha, \beta) \in \mathbb{R}^2$ *and* $\epsilon = \frac{1}{2} + it$ *such that*

$$\Re(p) = \int_{\mathbb{P}} \int_{\mathbb{R}} \left(\frac{p^2 + i^2}{4\alpha} \pm \beta \right) dp d\alpha = \pi(N) + \epsilon. \tag{13}$$

For instance

$\pi(N)$			$\Longleftrightarrow$			$\Re(p)-\epsilon$		
4	$=$	$\pi(10)$	$=$	0	$+$	10^2-1	/	$2^2.5$
25	$=$	$\pi\left(10^2\right)$	$=$	0	$+$	$10^{2^2}-1$	/	$2^2.97$
169	$=$	$\pi\left(10^3\right)$	$=$	0	$+$	$10^{3^2}-1$	/	$2^5.5^5.59$
1229	$=$	$\pi\left(10^4\right)$	$=$	-2	$+$	$10^{4^2}-1$	/	$2^{33}.3^3.5.7$
9592	$=$	$\pi\left(10^5\right)$	$=$	-88	$+$	$10^{5^2}-1$	/	$2^{67}.7$

(14)

You can obtain a very computational set with a discrete algorithm and improve the results by yourself such that $\beta=0$ for all $\pi(N)$ but, keep in mind that $\Re(p)$ holds for p prime.

The precise number of primes less than n is given by the following:

Algorithm 6. Code Mathematica

```
PrimePi[n]
```

1. PROVED
2. Does your equation embeds a prime counting function for a given quantity?

YES!

■

5.3 THE THIRD FLOOR

In our previous articles, we shown that the distribution of the primes depends on their last digit and how to build an infinite tree of primes from any given odd number. We have also proved that every prime number has it's own satellite which is also prime. Those results deserve to be considered as the most important breakthrough in number theory. In this section, we will complete the proof of the Next Prime theorem. This tightrope walker's step is obviously the most difficult because, we are looking for an equation which is able to generate a prime number without any computational complexity and without any primality test protocol.

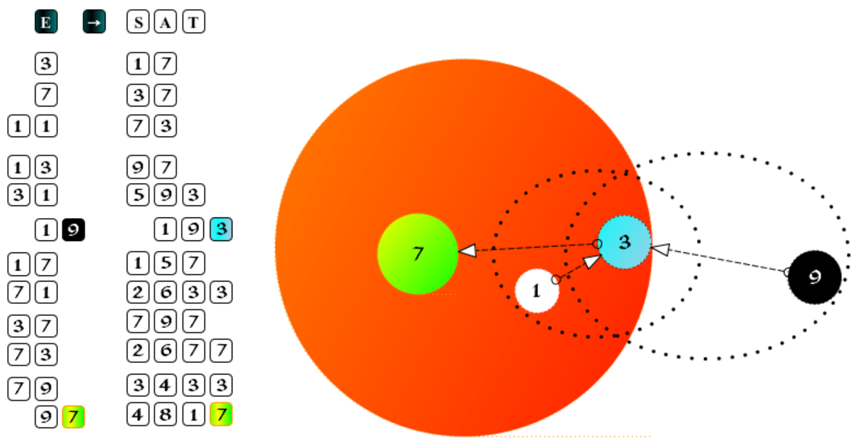
First, let $n=pq$ where p and q are primes and α as any factor of the odd number n and $\beta\equiv 5 \pmod{10}$ such that

$$\frac{n^2+(\beta\alpha)^2}{2\alpha^2}=\mathfrak{Prime}. \tag{15}$$

Now if we replace n by any given prime p such that $\alpha=1$, we obtain the following **iteration primes** partition:

$$\begin{array}{cccc} 3 & 7 & 11 & 19 \\ 17 & 37 & 73 & 193 \\ 317 & 137 & 173 & 1193 \\ 617 & 337 & 373 & 1693 \end{array} \tag{16}$$

There are specific conditions for which you will not obtain a prime. For instance, when $p = 3$ and $\beta = 15$, simply because β is divisible par 3 and so on. Otherwise, as we saw it your constellation of primes depends on the last digit of your original prime or odd number. We can observe this phenomena in the four basis of the DNA as well as in some planetary systems or in the Quantum Fields, as illustrate hereinafter:



Now, let us see the connections with our original equation. To fulfil the purpose in view and complete the proof, we merge the both equations such that

$$\frac{n^2 + (\beta\alpha)^2}{2 - \dfrac{2}{n^s}} = O. \tag{17}$$

For instance, let $\beta = 5$ and $s = 7$, according to the scale of n. We assume: O *will always contain the satellite prime number of* α. Recall that α is a factor of n. This is why we call such numbers the Euler Satellites or **E-Sat** as mentioned in the figure above, because the proof is given by means of the Euler product. For instance, let $n = 187$, we have

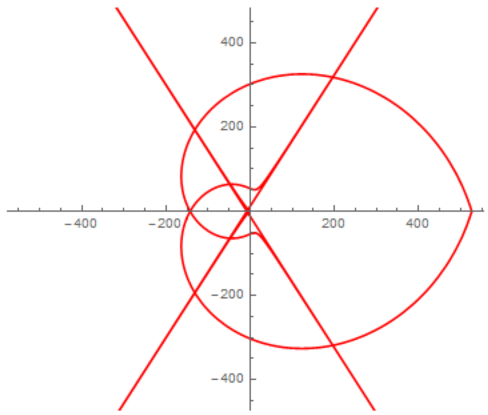
$$\frac{187^2 + (5.11)^2}{2 - \dfrac{2}{187^7}} = 11^2.157 \tag{18}$$

and

$$\frac{187^2 + (5.17)^2}{2 - \dfrac{2}{187^7}} = 17^2.73 \tag{19}$$

As you can see it $\left(11^2 + 5^2\right)/2 = 73$ and $\left(17^2 + 5^2\right)/2 = 157$, which means that the prime satellite of 11 will always be 73 as well as the E-Sat of 17 remains

157, which is very useful for factoring. If we plot the Euler-Sat, we observe the function of the primes as Gauss had predicted. In the polar below, you can see how this function is leaded by the complex number s. Indeed, if s is even you will obtain a perfect cross, otherwise, you will obtain a single line. With $s = 2$, we obtain a fancy logo for a brilliant Polytechnician.



Algorithm 7. Code Mathematica

```
PolarPlot[(tˆ2+(5t)ˆ2)/(2-(2/(tˆ2))), {t,−2π,2π},
PlotStyle->{Red,Directive[Dashed,Green,Orange]}, PlotRange-> All]
```

As a side note, we would like to say something much more relevant to the debate today. I have discovered that the distribution of the primes depends on their last digit when I was 12, but, let us recall that, such conjectures, which are engrossing puzzles, have intrigued scientists for over $3,000$ years. Indeed, proving the Next Prime theorem is equivalent to prove the prime number theorem and the infinitude of the primes, all at once. In the other hand, we recall that counting the number of primes is totally trite. Namely, if your proof of the prime number conjecture doesn't contain the next prime, sorry but you didn't. You had just brought to us another primes statistic report.

1. PROVED

2. PROVED

3. Does your equation generates the next prime?

YES!

■

5.4 THE DUPLEX

This floor is the last step of our journey with B. Riemann, through the sketch of the proof, because as we said, the purpose of the points 4 and 5 is to hone the generalization of the RH, see **Section 4**.

Therefore, if we reduce all the terms of our original equation, with some insightful sophistications, we obtain the **one sentence proof**

$$s\int_{-\infty}^{+\infty}\left(\frac{1}{2}+\frac{1}{i^2+\log(p)^{\pi}}\right)ds=\zeta(s), \tag{20}$$

where $i^2=-1$, $s\in\mathbb{C}$ and $p\in\mathbb{P}$.

Whereupon, we can observe immediately that the symmetry depends solely on the complex number s. Otherwise, since we are not interested by $\zeta(0)$, then, to obtain a steady equation, we can use multiple layouts to toggle s, pinning it in the right place, as shown below. Also, we generalize the real part such that, we assume: *for every prime p, there are infinitely many complex numbers s such that*

$$\int_{\mathbb{C}}\int_{\mathbb{N}}\int_{\mathbb{P}}\left(\frac{1}{n}+\frac{1}{i^2+\log(p)^{s\pi}}\right)dsdndp=\zeta(s), \tag{21}$$

where $i^2=-1$, $n\geq 1$, $s\in\mathbb{C}$ and $p\in\mathbb{P}$.

Also, to extend the real part, if you want, you can add another parameter like n^x.

Ultimately, if you think that the equation above is straightforward, so you didn't understand. Indeed, a pilot steers the most advanced and sophisticated jet fighter by means of a simple joystick control. Therefore, this equation maybe thought as of the perfect joystick to wield the complex Riemann zeta function.

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6 THE PROOF REPORT

1. PROVED
2. PROVED
3. PROVED
4. PROVED
5. PROVED

The Edifice of the Proof is now complete.:

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7 THE CONSEQUENCES OF THE RH PROOF

The discussion then boils down to how the harvest solution of the Riemann Hypothesis implies the proof or disproves other related conjectures, like a Domino.

7.1 THE ABC THEOREM

The genuine formulation of the abc-conjecture is: *for every $\epsilon > 0$, there are only finitely many triples of coprime positive integers $a + b = c$ such that $c > d^{1+\epsilon}$, where d denotes the product of the distinct prime factors of abc.* Any other formulation of the abc conjecture is a total nonsense.

Theorem. *There doesn't exist any triple of coprime positive integers $a+b = c$ such that $c > d^{1+\epsilon}$.*

The one sentence Proof.

$$\frac{\log\left(\pi^{c}\right)}{\log\left(\sqrt{\pi^{2abc}}\right)} = d^{1+\epsilon} = \frac{1}{ac} + \frac{1}{bc} < 1 < c. \tag{22}$$

■

After the publication of the article *"Stealth Elliptic Curves and the Quantum Fields"*, in 2013, the University of Leiden has stoped the **abc@home** project, whilst, some people were trying to reformulate the abc conjecture. Also, it was quite interesting to observe the sparkling and the media turnmoil around the Terence Tao and Shinichi Mochizuki productions. We assume, none got through and none of them actually cleared up the problem. In the other side, the math community was utterly aware about this issue. For instance, you can see **here** that the question has been discussed on wikipedia but, mysteriously, some people (who are they?) decided to hide the proof and to delete from the Internet any related **link**. Therefore, I submitted the proof to the ICM and the complete abstract was accepted by the organizing committee of the International Congress of Mathematicians, as the Keynote part of the short communications at Coex in Seoul 2014. This is a glimpse of the storytelling, see **here**.

7.1.1 RIEMANN AND THE ABC THEOREM

In this subsection, we will release two valuable proofs of the Riemann Hypothesis by means of the abc theorem. Recall: *there are infinitely many nontrivial zeros and they all have real part $1/2$.*

The one sentence Proof.

$$\prod_{d^{1+\epsilon}}^{\infty}\left(\frac{1}{2} + \frac{\log\left(\pi^{c}\right)}{\log\left(\sqrt{\pi^{2abc}}\right)}\right) s = \zeta\left(s\right). \tag{23}$$

We recall: For every (a,b,c) triples, there exists a quality $1 < \underset{\dot{a}\ddot{b}\dot{c}}{Q} < 2$ *such that*

$$\frac{\log\left(\pi^{c}\right)}{\log\left(\sqrt{\pi^{ab}}\right)} = \underset{\dot{a}\ddot{b}\dot{c}}{Q} = 1 + \frac{\log\left(\pi^{c}\right)}{\log\left(\sqrt{\pi^{bc}}\right)}. \tag{24}$$

If $a = 1$*, then replace* π^{ab} *by* π^{2ab} *and* π^{bc} *by* π^{2bc}.

For the second time. We assume: *there are infinitely many nontrivial zeros and they all have real part* $1/2$.

The one sentence Proof.

$$\prod_{\underset{\dot{a}\ddot{b}\dot{c}}{Q}}^{\infty}\left(\frac{1}{2} + \frac{\log\left(\pi^{c}\right)}{\log\left(\sqrt{\pi^{bc}}\right)}\right) s = \zeta\left(s\right). \tag{25}$$

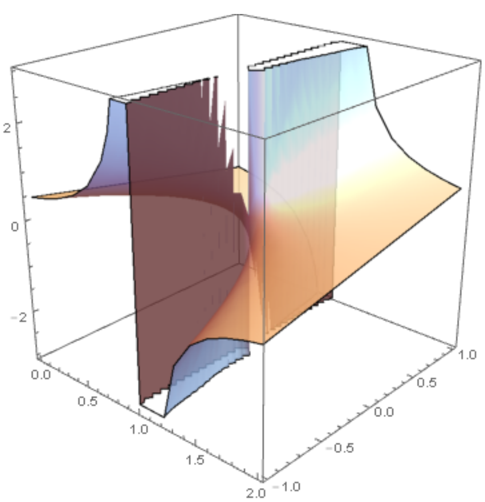
For more details see: "Stealth Elliptic Curves and the Quantum Fields.

To complete the proof we merge the both equations such that

$$\prod_{\underset{\dot{a}\ddot{b}\dot{c}}{Q}}^{\infty}\left(\frac{1}{2} + \frac{\log\left(\pi^{c}\right)}{\log\left(\sqrt{\pi^{bc}}\right)}\right) s = \zeta\left(s\right) = \prod_{k=1}^{\infty}\frac{s}{2\left(p_k + i^2\right)}, \tag{26}$$

where $i^2 = -1$*,* $s \in \mathbb{C}$ *and* $p_k \in \mathbb{P}$ *denotes the k-nth prime.*

For more details about the Right Hand Side, please, see the **Section 5**. The easiest way to visualize the connections between the abc theorem and the Riemann Hypothesis is to plot $\frac{c}{2b}$ and $\frac{s}{2\left(p + i^2\right)}$ such that



$\blacksquare$

7.2 THE TWIN PRIME THEOREM

Theorem. *There are infinitely many odd twin primes P satisfying*

$$\liminf_{i\to\infty} \frac{P}{\sum\limits_{\substack{p_i>2 \text{ with } p_i-p_j\geq 2 \\ \text{and } i\neq j}} p_i} = \Re_k, \tag{27}$$

where $\Re_k$ denotes the Riemann constant $1/2$.
The one sentence Proof.

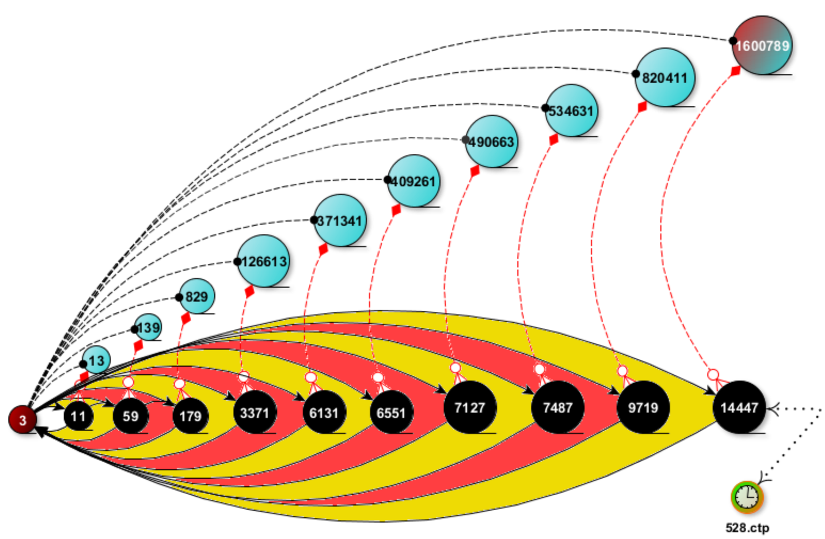
$$\prod_{k=1}^{\infty} \frac{P_k}{\sum\limits_{\substack{p_i>2 \text{ with } p_i-p_j\geq 2 \\ \text{and } i\neq j}} p_i} = 0. \tag{28}$$

Therefore, we observe that all the twin primes fetch the critical line. Finally, to complete the proof, we assume: *the numerator p will always be the second twin of the given prime pair such that $p-2$ is prime, which proves clearly that the smallest gap between consecutive primes corresponds to 2 such that*

$$\liminf_{n\to\infty} (p_{n+1} - p_n) = 2, \tag{29}$$

where p_n denotes the n-th prime.

For more details, see the article "*RSA-T. The Oval Pylon*". We have computed the Riemann Twin Primes or the **Royal Primes** over 1.5312564×10^7. Here we get an excerpt:



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A side note. We will not dwell on that issue. Nevertheless, for aught I know, Yitang Zhang didn't solve anything but suddenly, he was propelled into the spotlight by an orchestrated massive propaganda campaign fostered by who? The conjecture is clear enough, to solve the twin prime problem you have to prove that for infinitely many primes, $p-2$ will always be a prime, which corresponds to the smallest gap between twin primes. But, please, don't waste our time with finitely many pigeons (Bird Strike). Indeed, The rationale of Y. Zhang's article seems rather specious and rotten, in trying to justify a haphazard primes gap. Finally, we shall be suggesting to our colleagues that we hold harvest discussions to achieve genuine equality.

7.2.1 RIEMANN AND THE TWIN PRIME THEOREM

Through this Subsubsection, we pay tribute to the Great British Mathematicians G. H. Hardy and John Littlewood who are the pathfinders, about the connections between the Riemann Hypothesis and the Twin Prime conjecture. Indeed, we assume: *the first Hardy-Littlewood conjecture is equivalent to*

$$\prod_{\substack{p\in\mathbb{P}\\ p\geq 3}}^{\infty}\left(1-\frac{1}{\left(p+i^2\right)^2}\right)\approx C_2\approx \lim\sum_{n=1}^{\infty}\frac{1}{(2n+2)^{\frac{1}{4}(1+n^2)}},\tag{30}$$

where $C_2\approx 0.6$ denotes the twin prime constant and $i^2=-1$.

What's more, we have shown in the previous article: "*The Riemann Hypothesis Proof and the Quadrivium Theory*", how the last digit and some fractions play an important role in the distribution of the primes. As for the connections between the RH and the Hardy-Littlewood conjecture we can observe

$$\prod_{\substack{p\in\mathbb{P}\\ p\geq 3}}^{\infty}\left(1-\frac{1}{\left(p+i^2\right)^2}\right)\approx\frac{\pi}{5}\approx\frac{1}{\frac{2}{3}+\frac{1}{6}+\frac{1}{2}+\frac{1}{3}}.\tag{31}$$

Note: 2111 and 3623 are primes. Every fraction generates a specific "class" of primes, according to their last digit. Finally, to complete the proof, we generalized the RHS such that, we assume:

Theorem. *The smallest gap between consecutive primes corresponds to* 2.

Proof.

$$\liminf_{n\to\infty}\left(p_{n+1}-p_n\right)=2=\lim_{n\to\infty}\frac{1}{\frac{1}{2}+\frac{7}{2p_n}},\tag{32}$$

where p_n denotes the n-th prime. You can replace 7 *by any constant* $k>0$.

Namely, this maybe considered as an important result in the history of the resolution of the Riemann Hypothesis. For more details about the Right Hand Side, please, see the **Section 5**.

■

7.3 THE LEGENDRE CONJECTURE PROOF

7.3.1 THE FORMULATION

Adrien-Marie Legendre $(1752 - 1833)$ is a brilliant French Mathematician, who deserves an outstanding tribute, for having formulated the most important problem in number theory, after the Riemann Hypothesis. We will try to prove why. So far, more than two hundred years ago (i.e. 1798, according to M. Desboves), Adrien-Marie Legendre had conjuctured that*: there exists, at least, a prime between n^2 and $(n+1)^2$ for any $n \geq 1$.*

7.3.2 LEGENDRE AND THE TOP GUN REPORT

In number theory, simple is the neatest word to define the complexity. By the way, two weeks ago, a pilot asked me a tough question: "*During a battle, if I lose the control of my navigation instruments, how can I find the nearest airport, according to a given amount of fuel and without any computational complexity?*". Obviously, the answer would be: "*Ask Google Map! Do you copy Maverick?*". For crafty mathematicians, such interesting questions, even if the mileage may vary, fetch the Legendre conjecture. So, I spent exactly one hour on this issue, which may be considered as a critical time situation for a flying pilot with a minimum amount of fuel. In this Subsubsection, we will elaborate the sketch of the proof by starting from the end. Indeed, the main purpose behind the Legendre conjecture is to generalize the prime number theorem and to solve the next prime conjecture. Therefore, follow up to Gauss, let us define $\pi(N)$ as the number of primes less than n . We assume:

Theorem. *For every $n \geq 1$, there exists $\{(\alpha,\beta) \in \mathbb{R}^2 : \log(2)^{\pi\beta} \neq n\alpha\}$ and $\epsilon > 0$ such that*

$$\int_{\mathbb{R}} \frac{n\,(n+3)}{2\,(n\alpha \pm \log(2^{\beta\pi}))} d\alpha = \pi(N) + \epsilon. \tag{33}$$

∎

7.3.3 RIEMANN AND THE LEGENDRE THEOREM

Recall: *there are infinitely many nontrivial zeros and they all have real part* ½.

The one sentence Proof.

$$s \int_{\mathbb{C}} \int_{\mathbb{N}} \int_{\mathbb{R}} \frac{n\,(n+3)}{2\,(n\alpha \pm \log(2^{\beta\pi}))} ds dn d\alpha = \zeta(s)\,, \tag{34}$$

where $(\alpha,\beta) \in \mathbb{R}^2$, $s \in \mathbb{C}$ *and* $n \geq 1$.

Finally, we can observe the perfect symmetry as well as we can generalize the real part.

∎

To compute $\pi(N)$ over 10^9 with the Legendre set and to obtain a time machine precision, we define $\pi = 3.14159$ such that

Algorithm 8. Code Mathematica

INPUT	***OUTPUT***
Solve[Legendre ==4 && y=2 && n==10,{x}]	x -> 1.625 + ϵ
Solve[Legendre ==25 && y=2 && n==10ˆ2,{x}]	x -> 2.06 + ϵ
Solve[Legendre ==168 && y=17 && n==10ˆ3,{x}]	x -> 2.985119047619048 + ϵ
Solve[Legendre ==1229 && y=325 && n==10ˆ4,{x}]	x -> 4.0695687550854345 + ϵ
Solve[Legendre ==9592 && y=325 && n==10ˆ5,{x}]	x -> 5.212833611342786 + ϵ
Solve[Legendre ==78498 && y=325 && n==10ˆ6,{x}]	x -> 6.369608142882621 + ϵ
Solve[Legendre ==664579 && y=325 && n==10ˆ7,{x}]	x -> 7.523562285296405 + ϵ
Solve[Legendre ==5761455 && y=325 && n==10ˆ8,{x}]	x -> 8.678363625160658 + ϵ
Solve[Legendre ==50847534 && y=325 && n==10ˆ9,{x}]	x -> 9.833318593188807 + ϵ

Ultimately, we can observe in the partition above how the Legendre set is useful to determine the precise number of primes less than a given quantity.

■

Now let us elaborate the reel. We assume, for every $n \geq 1$, there exists $\alpha \in \mathbb{R}$ such that

$$n^2 < \underbrace{n(n+1) \pm \alpha}_{\mathfrak{Prime}} < (n+1)^2 . \tag{35}$$

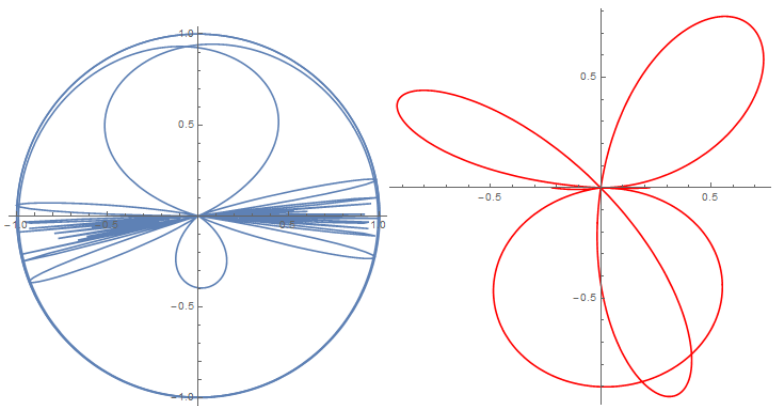
In the general case $\alpha = \pm 1$ and for some special cases $\alpha = \pm \left(\underbrace{(n+1)^2 - n^2}_{\mathfrak{Prime}} \right)$. We can improve this relation such that

$$n^2 < \underbrace{n^2 + 3n \pm \alpha}_{\mathfrak{Prime}} < (n+1)^2 . \tag{36}$$

This harvest solution is very computational since it fails only for some specific cases, which you can predict in your algorithm. For instance, you will not obtain a prime number, if $n^2 + 3n$ is mod $\{0, 4, 8, \ldots\}$ and so on. Otherwise, we can observe that there exists an homogeneous linear recurrence relation which generates what has to be called the **Legendre numbers,** as well as Fibonacci has his own. The first sequences are $\{2, 5, 9, 14, 20, 27, 35, 44, 54, 65, 77, \ldots\}$, which can be generalized as follows:

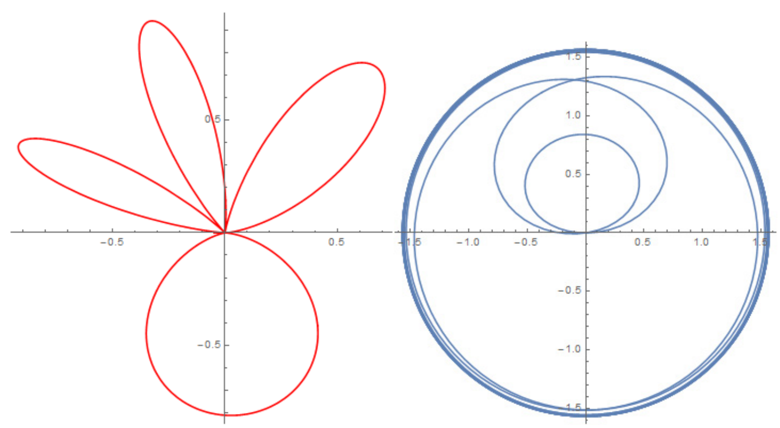
$$a_n = \frac{1}{2}\left(n^2 + 3n\right) . \tag{37}$$

We can observe hereinafter the deep connections with the nontrivial zeros of the Riemann zeta function.



Algorithm 9 & **10**. Code Mathematica

PolarPlot[Cos[2/Abs[t (3 + t)]]^2, {t,-4π,4π}]
&
PolarPlot[Sin[Abs[t (3 + t)]/2], {t, $-\pi$, π}, PlotStyle->Red]



Algorithm 11 & **12**. Code Mathematica

PolarPlot[Sin[Abs[t (3 + t)]/2]^2, {t, $-\pi$, π}, PlotStyle->Red]
&
PolarPlot[ArcTan[(θ (3 + θ))/2], {θ, -100, 100}]

Ultimately, if we merge the both relations we obtain $\pi(N)$.

■

The first consequence is related to the Goldbach conjecture since there exists an embedded Goldbach function on the form

$$\underset{n\to+\infty}{G(n)} = 2\,(n+2) \equiv 0 \pmod 2, \tag{38}$$

which is, indeed, the simplest formulation of

$$(n+1)^2 + 3\,(n+1) - (n\,(n+3))\,. \tag{39}$$

Thereupon, one can see that for some cases $\pi(N)$ can be expressed as follows:

$$\pi(N) \approx \frac{G\,(n)}{?}. \tag{40}$$

The second consequence, we assume: *there are infinitely many primes which can be expressed as the sum of two squares such that*

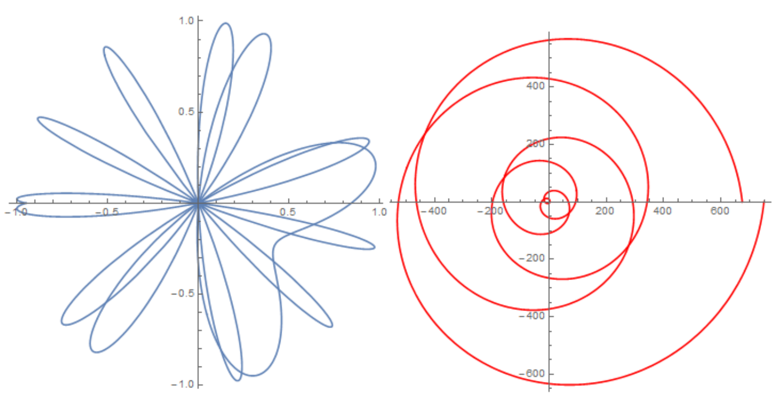
$$n^2 + (n+1)^2 = \mathfrak{Prime}. \tag{41}$$

This set of primes is very computational since it generates only primes ended by $\{1, 3\}$, otherwise the prime lies in $\equiv 0 \pmod 5$. Also, the distribution of the prime numbers follows a very precise pattern. Indeed, you will never obtain two consecutive primes ended by 3. You can generate the first sequences $\{\{n->1, p->5\}, \{n->2, p->13\}, \{\ldots\}, \{n->974, p->1899301\}\}$ with the following algorithm.

Algorithm 13. Code Mathematica

```
Solve[n^2+(n+1)^2==p && Element[p,Primes] && 0<n<10^3, {n,p}, Integers]
```

We can observe the perfect illustration by the following patterns:



Algorithm 14 & **15**. Code Mathematica

```
PolarPlot[Sin[Abs[t^2 + (1 + t)^2]], {t, −π, π}]
&
PolarPlot[t^2 + (1 + t)^2, {t, −6π, 6π}, PlotStyle->Red]
```

7.3.4 THE PENULTIMATE PRIME THEOREM

We are now well poised to elaborate the leer of the proof. Indeed, to settle completely the Legendre conjecture, we will articulate the sketch of the proof around three steady points. Above all else, we will determine whether there exists an odd number between n^2 and $(n+1)^2$ for every $n \geq 1$. In the second time, we will prove that there exists at least a prime in the given interval. At last, we will see how to obtain the previous or the next prime for any given interval.

Step 1.

Theorem. *For every $n \geq 1$, there exists at least an odd number α between n^2 and $(n+1)^2$.*

*Proof. If $(n+1)^2 \equiv 0 \pmod 2$, then there exists $\beta \equiv 1 \pmod 2$, and **vice versa**, such that with $\beta \leq (n+1)^2 - n^2$*

$$(n+1)^2 - \beta = \alpha, \text{ and } \underbrace{\frac{5}{2}(1+(-1)^{\alpha})}_{\alpha \text{ odd}=0}. \tag{42}$$

Step 2.1. Honestly, there are many different ways to solve this issue. Nevertheless, we will introduce two elegant solutions, among others.

Theorem. *For every $n \geq 1$, there exists at least an odd prime p between n^2 and $(n+1)^2$.*

Proof.

$$\Omega \pm \alpha = \delta, \tag{43}$$

where α is any integer, $\delta \equiv 0 \pmod p$, and Ω denotes the product of n^2 and $(n+1)^2$. For instance

n	**p**	$\mathbf{(n+1)^2}$	$\mathbf{\Omega} - \alpha = \delta$
1	$2, 3$	4	$\Omega - 1^2 = 3$
2	$5, 7$	9	$\Omega - 1^2 = 5.7$
3	$11, 13$	16	$\Omega - 1^2 = 11.13$
4	$17, 19, 23$	25	$\Omega - 3^2 = 17.23$
5	$29, 31$	36	$\Omega - 1^2 = 29.31$
6	$37, 41, 43, 47$	49	$\Omega - 1^2 = 41.43$
″	″	″	$\Omega - 5^2 = 37.47$
7	$53, 59, 61$	64	$\Omega - 3^2 = 53.59$
8	$67, 71, 73, 79$	81	$\Omega - 1^2 = 71.73$

(44)

As we can see it in the partition above, most of the time α is a square and $\delta = pq$ where p or q lies between n^2 and $(n+1)^2$. In cryptography, this is very useful for factoring. Moreover, you can build an infinite set of twin primes. Ultimately, we proved with the first sequence, with $\sqrt{\Omega - 3} = 1$, that 3 is the penultimate prime between n^2 and $(n+1)^2$.

Step 2.2. For the second time.

Theorem. *For every $n \geq 1$, there exists at least an odd prime p between n^2 and $(n+1)^2$.*

Proof.

$$\lim_{\alpha \to \pm\infty} \frac{n^3 (n+1)^4}{\alpha^2} = e^{i\pi} = -1, \tag{45}$$

where the integer $\alpha \neq 0$.

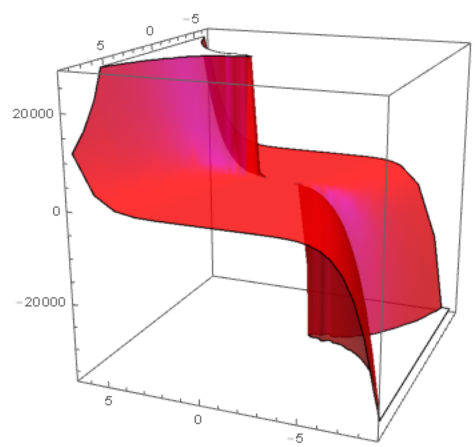
Nevertheless, to elaborate the proof, we will consider the equivalent version, which is slightly different, but more computational. It follows, with $n \geq 2$

$$\int \frac{n^4 (n+1)^4 - n\alpha^2}{n\alpha^2} d\alpha = \mathfrak{Prime} + \epsilon. \tag{46}$$

Note that $n^2 + n = \alpha$ corresponds to the threshold from which you can dive into the interval and find the primes between n^2 and $(n+1)^2$. Moreover, according to α, you can generate at least a prime less than n, as well as the next prime. Ultimately, we can observe the deep connections between ϵ and the nontrivial zeros of the Riemann zeta function. For instance

$$7^2 < \frac{7^3 (7+1)^4}{151^2} = 61 + \epsilon < (7+1)^2. \tag{47}$$

With $\alpha = 144$ you obtain $67 + \epsilon$ which is the next prime. We can illustrate the situation with the Royal Throne hereinafter.



Algorithm 16. Code Mathematica

```
Plot3D[(n^4 (1 + n)^4 - n x^2)/(n x^2), {n, -8.6, 6.5}, {x, -6.5, 8.6}, MeshStyle->None,
PlotStyle->Directive[Red,Specularity[Blue,20],Opacity[0.8]],
ExclusionsStyle->{None,Red}, BoxRatios->{1, 1, 1}]
```

Step 3.1. The Next Prime

Theorem. *For every $n \geq 1$, there exists $\beta \equiv 5 \pmod{10}$ and α integer such that, at least an odd prime p lies between n^2 and $(n+1)^2$.*

Proof.

$$\alpha\beta \pm n^2 (n+1)^2 \equiv 0 \pmod{p}. \tag{48}$$

For instance

$$\begin{array}{l} 2.5 - 4 = 2.3 \\ 67.85 - 8^2.9^2 = 7.73 \\ 71.425 - 8^2.9^2 = 67.373 \end{array} \tag{49}$$

where $1 < 2, 3 < 4$ and $8^2 < 67, 71, 73, 79 < 9^2$. To obtain the Next Prime, you have to shift β, like a gear, such that for instance

$$3\beta - 4 \Leftrightarrow 11, 41, 71, 101, 131, 191, \ldots, 911, \ldots, 2861, \ldots \tag{50}$$

As you can see it, the distribution of the primes depends on their last digit. Also, there are some exceptions for which you will not obtain a prime. For example if β or the output is divisible by a previous prime. The set is very computational. You can use the following to generate the first sequence $< 10^3$.

Algorithm 17. Code Mathematica

```
Solve[3β-4==p && Element[p,Primes] && Mod[β,5]==0 && 1<β<10^3, {β,p}, Integers]
```

Step 3.2. Now, for the generalization, we introduce the Adrien-Marie Legendre modular constellation of primes on the form

$$kab - a^2 + b^2 \equiv 0 \pmod{c}, \tag{51}$$

where, $b - a = 1$ and $a + b \equiv 0 \pmod{c}$ as the roots of the Legendre Next Primes function and $k = 3$. So, from $a = 2$ and $b = 3$ we have

$$\begin{array}{ccccc} 3.2.3 & - & 2^2 + 3^2 & = & 5 \\ 3.7.8 & - & 7^2 + 8^2 & = & 5.11 \\ 3.12.13 & - & 12^2 + 13^2 & = & 5.31 \\ 3.17.18 & - & 17^2 + 18^2 & = & 5.61 \\ 3.22.23 & - & 22^2 + 23^2 & = & 5.101 \\ 3.27.28 & - & 27^2 + 28^2 & = & 5.151 \\ 3.32.33 & - & 32^2 + 33^2 & = & 5.211 \end{array} \tag{52}$$

Algorithm 18. Code Mathematica

```
Solve[(3ab)-(a^2+b^2)==5z && a+b==c && b-a==1 && a<b && Mod[c,5]==0 && Ele-
ment[z,Primes] && 1<a<50 && 1<b<50 && 1<z, {a,b,c,z}, Integers]
```

You can change the core parameters, to obtain another modular distribution of primes with another last digit. Ultimately, we enhanced completely the proof, *ultra-petita*.

■

8 MISCELLANEOUS

8.1 ABOUT π

The Euler–Mascheroni constant.

In the movie "*Hidden Figures*", did Kevin Costner said: "*here at NASA, we all π the same color?*" It's a nice touch. Isn'it? In French the prononciation is a little bit confusing, but what would be the color of π? There is an interesting question whether the Euler–Mascheroni constant γ is rational? We assume: *there exists $\chi \in \mathbb{R}$ and $\epsilon > 0$ such that*

$$\pi \approx \frac{\sqrt{(\chi+\epsilon)^{\gamma}}}{\gamma}, \tag{53}$$

where, for instance $\gamma \approx 0,5772156649$ and $\chi + \epsilon \approx 7,86386724$. Which means that the Euler–Mascheroni constant may not be rational. But, this is not a complete proof.

The Golden Ratio.

We assume there exists a complex number s such that

$$\frac{1+\sqrt{5}}{2} \approx 1 + \frac{\sqrt{3}}{\pi^{s}}, \tag{54}$$

where $s \approx 0,9002280645$.

8.2 PALINDROMIC PRIMES + SPECIAL NUMBERS

Last week, my daughter **Wurus**, who is 5 years old, said that computing is boring because, over 10 all numbers are similar. Therefore, she asked me to nudge her (i.e., R. Thaler and C. Sunstein) how to compute the most special number with her App. So, I asked back: "*why?*". She said that she wants to teach something new to her fellows. First, I recall that: *"you are rather supposed to learn at school, not to teach!"*. Whilst, I asked her: *"what a special number would look like?"*. She said: *"a number which grows from the left and decreases towards the right or a number which grows from the middle with the same edges"*. *"Shrug"*, I replied. After 5 minutes, I realized how hard is the problem. Indeed, it's beyond the classical recreational Mathematics. On the Internet, it turns out that the largest palindromic prime, as of Nov 2014, has 474501 digits. By the way, many Great Mathematicians have been working on such questions. Here we are.

Theorem. *There are infinitely many integers on the form*

$$2\Theta^2 - 1 - \Delta, \tag{55}$$

where Θ is any natural repdigit and Δ denotes the adjustment factor.

For instance

$$2\left(3333^2\right) - 1 - 19996000 = 2221777. \tag{56}$$

8.2.1 THE WURUS NUMBER

Wurus means gold, but the Wurus number is not the Golden Ratio. The Wurus number is an **Ambidextrous** number which swells from the both edges, in a perfect symmetry. So, to obtain the mirror of the infnity, we define the generalization of the Wurus number as follows:

$$(\Theta + 1)^2 - k\Theta - \Delta, \tag{57}$$

where Θ is a repunit number, k any constant and Δ denotes the adjustment factor.

For instance, with $k = 3$, $\Theta = \underbrace{1111111111}_{10}$ and $\Delta = 1111111109888888890$

$$\mathfrak{Wurus} = 123456789987654321. \tag{58}$$

The prime factors are

$$\mathfrak{Wurus} = 3^2.11.37.41.271.9091.333667. \tag{59}$$

Now, we know how to raise a number from the middle while keeping the same edges, with a deft hand. This is completely new in number theory.

For the music of the primes: *There Is Something New Under The Sun.*

MEMENTO MORI

★

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